Heat and mass transfer from rotating cones

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Heat transfer by convection from isothermal rotating cones is investigated experimentally by measuring the sublimation rate from naphthalene-coated cones and using the analogy between heat and mass transfer. Measurements are made for a range of conditions from entirely laminar flow to conditions when the outer 70 % of the surface area is covered by turbulent flow. Mass-transfer measurements for laminar flow over cones of vertex angles 180°, 150°, 120° and 90° are in good agreement with the theoretical prediction. For turbulent flow, experimental results for cones of the above vertex angles also agree very well with the semi-empirical analogy calculations for the disk case. A different heat- and mass-transfer relationship with the rotational Reynolds number is observed in the measurements on the 60° cone, and is believed to be due to a change of flow characteristics. The instability and the transition of flows over different cone models are also discussed.

Introduction

The flow induced by a disk (i.e. a cone with vertex angle of 180°) rotating in its own plane was first studied by von Kármán (1921). Numerical results were later given by Cochran (1934) for laminar flow and by Goldstein (1935) for turbulent flow. The solution in the laminar case is an exact one of the Navier–Stokes equations, and indicates a boundary-layer character near the disk surface. An approximate solution for the heat transfer by laminar flow from a rotating disk was first obtained by Wagner (1948). The exact solution of the laminar heat-transfer problem was given by Millsaps & Pohlhausen (1952). Their heat-transfer results would be correct, as pointed out by Cobb & Saunders (1956), if the specific heat at constant volume in their energy equation were replaced by the specific heat at constant pressure. Ostrach & Thornton (1958) have presented a critical review of works of different investigators. Neglecting the viscous dissipation, they obtained a heat-transfer result for a Prandtl number of 0·72. Sparrow & Gregg (1959) extended the calculation of heat-transfer coefficients for a wide range of Prandtl numbers.

For the turbulent heat transfer from a rotating disk, semi-empirical calculations based on the friction analogy were first carried out by Cobb & Saunders, and later refined by Kreith, Taylor & Chong (1959). The analogy results of Kreith *et al.* agree favourably with the experimental data. Davies (1959) has given an approximate solution for the limiting case when the disk surface is completely covered by turbulent flow. His solution agrees with the corresponding analogy results by Cobb & Saunders.

Experimentally, the heat transfer from rotating disks has been measured by several investigators. Heat-transfer measurements for a heated rotating disk have been reported by Young (1956) in the laminar flow region and by Cobb & Saunders in both the laminar and the turbulent regions. In these investigations, difficulties have been encountered in maintaining and measuring a uniform surface temperature or in accounting for unwanted heat losses. A different experimental scheme, which eliminated these difficulties, was employed by Kreith *et al.* Their experimental model was a naphthalene-coated disk, from which the rates of sublimation were measured. The measured mass-transfer rates from the rotating risk are related to the corresponding heat-transfer process by means of the analogy method. The rate of mass injection due to sublimation is so low that the process is equivalent to the heat-transfer process from an isothermal impervious disk. The mass-transfer results of Kreith *et al.* include results in both the laminar and the turbulent flow regions.

The laminar incompressible flow induced by a rotating cone was first studied analytically by Wu (1959). Under boundary-layer approximations, all the numerical results except those for pressure distributions were shown to be obtainable from the solutions for the rotating disk. The heat transfer in laminar flow about a rotating cone under boundary-layer approximations, as indicated by Tien (1960), can be also obtained from the corresponding results in the disk case. The boundary-layer assumption, as pointed out by Wu and Tien, restricts the application of their results to cones with sufficiently large vertex angles.

The purpose of this investigation is to determine experimentally the mass transfer, and, by analogy, the heat transfer from isothermal cones rotating in an infinite environment. The analogy method of Kreith *et al.* is used in this investigation. Measurements are made in both the laminar and the turbulent flow regions, and are compared to the predicted results.

Experimental arrangement

The rate of sublimation from various conical surfaces rotating in air was measured in this experiment. The sublimation models used consisted of various aluminium cones coated with naphthalene ($C_{10}H_8$). The cones were mounted on a vertical shaft, facing upward and rotating at constant speeds ranging from 570 to 9800 r.p.m. The mass loss due to sublimation while at the specified speed, the ambient temperature and the speed of rotation were obtained for each run. From this data, the average Sherwood number was calculated, and, by analogy, the average heat-transfer coefficient can be determined.

The vertex angles of the cones investigated were 180° (disk), 150°, 120°, 90° and 60°. The cones were $6\frac{1}{2}$ and 8 in. in diameter and were made to two designs. The disk, 150° and 120° cones were as pictured in figure 1. They were aluminium cones with a $\frac{1}{16}$ in. lip on the rim, and with circumferential grooves in the surface. The purpose of the lip and grooves was to hold the naphthalene coating to the aluminium cone. The 90° and 60° cones were similar in design, but included a projection of the aluminium portion of the cone through the naphthalene coating at the vertex. This was found necessary to provide adequate strength in this region for machining the naphthalene surface on a lathe. This aluminium vertex does not affect the flow of air, and has negligible surface area.

The mechanism for rotating the cones is shown in figure 1. This includes a $\frac{3}{4}$ h.p., three phase, 440 V a.c. electric motor connected to a vertical shaft by a V belt and pulleys. This drive arrangement was chosen because it offered a more constant speed than other methods available. The speed of rotation

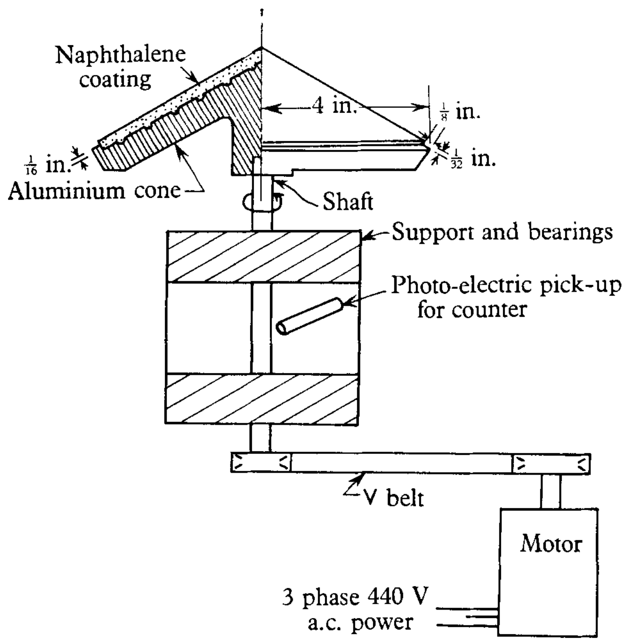

FIGURE 1. Sketch of the experimental equipment.

was measured with an electronic counter with a photo-electric pick-up device. This permitted the speeds of rotation to be measured accurately to within 1 % during rotation. The small additional load imposed by placing the cone on the shaft did not decrease the speed of rotation measurably. A mechanical counter and strobotac were used for comparison.

The experimental procedure and precautions follow closely those in the work of Kreith *et al.* and will not be repeated here.

The experimental values of average mass-transfer coefficient, $\bar{k}_c$, are obtained from the rate of mass transfer, m, through the following relation

$$\bar{k}_c = mR_v T_s / p_{vs} A,$$

where R_v is the gas constant of the naphthalene vapour, T_s the temperature at the rotating surface, p_{vs} the vapour pressure of the naphthalene at T_s and A the mass-transfer area. The physical properties of the naphthalene were taken from table 2 of the paper by Christian & Kezios (1957).

Mass-transfer results in laminar range

For the laminar flow induced by a rotating cone, the heat-transfer relation obtained by Tien under the boundary-layer assumption can be written as

$$\overline{Nu} \equiv (\bar{h}_c x_0/k) = C(\omega x_0^2 \sin\alpha/\nu)^{0\cdot5} \equiv C Re_0^{0\cdot5},$$

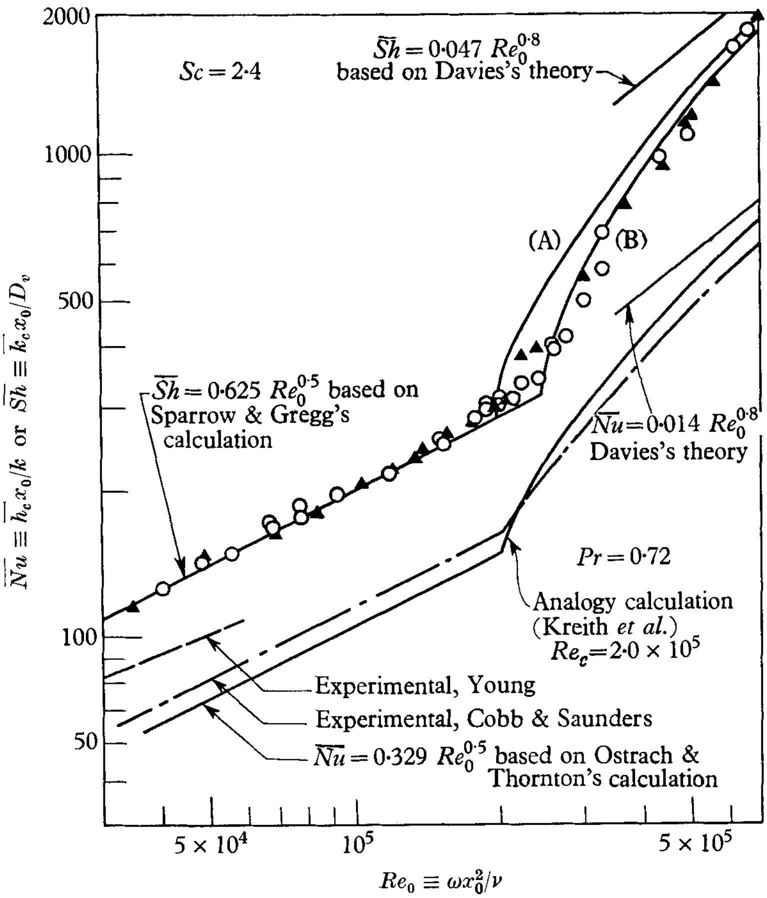

FIGURE 2. The variation of the average Sherwood and the average Nusselt number with the rotational Reynolds number for a rotating disk. ○ Experimental (present investigation); ▲ experimental (Kreith *et al.*). Analogy method (Kreith *et al.*): (A), $Re_c = 2{\cdot}0 \times 10^5$; (B), $Re_c = 2{\cdot}5 \times 10^5$.

where $\overline{Nu}$ and $\bar{h}_c$ are the average Nusselt number and the average heat-transfer coefficient over the entire heat-transfer area, x_0 the slant height, 2α the vertex angle and ω the angular velocity of the cone, k the thermal conductivity and ν the kinematic viscosity of the ambient fluid, C a constant dependent only on the Prandtl number, and Re_0 the rotational Reynolds number. For a Prandtl number of 0·72, the latest calculation by Ostrach & Thornton gives C a value of 0·329 instead of 0·335 given by Wagner and 0·28 by Millsaps & Pohlhausen. For the case of a rotating disk the predicted results are shown in figure 2 along with the experimental heat-transfer results by Young and by Cobb & Saunders. The experimental values indicate larger heat-transfer coefficients than predicted.

Based on the analogy between heat and mass transfer, the corresponding mass-transfer relation for the laminar flow over a rotating cone is given as

$$\overline{Sh} = \overline{k}_c x_0/D_v = C Re_0^{0\cdot 5},$$

where $\overline{Sh}$ is the average Sherwood number over the whole mass-transfer area and D_v the mass diffusivity. The constant, C, in this case is a function of the Schmidt number alone. The average temperature in the present investigation

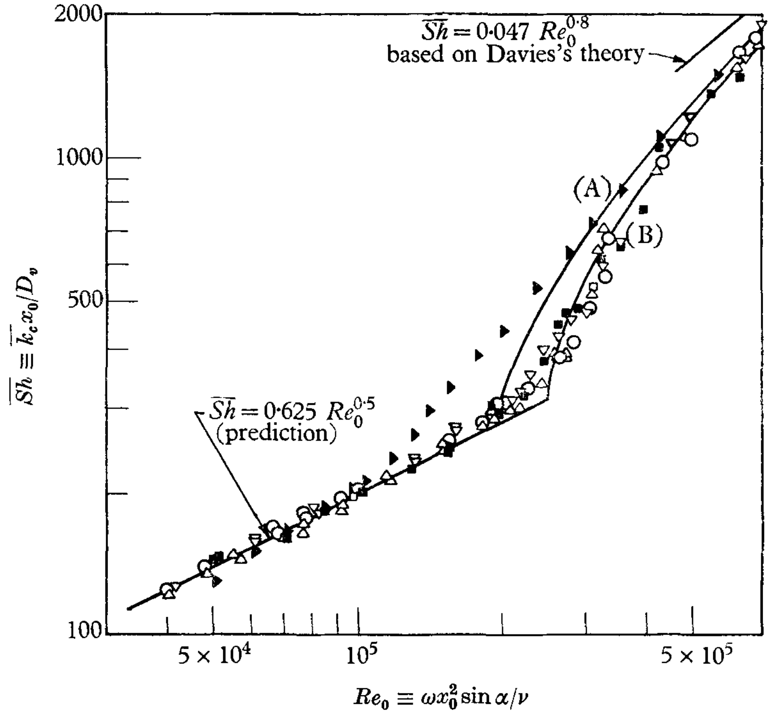

FIGURE 3. The variation of the average Sherwood number with the rotational Reynolds number for rotating cones. Analogy results for disk (Kreith *et al.*): (A), $Re_c = 2{\cdot}0 \times 10^5$; (B), $Re_c = 2{\cdot}5 \times 10^5$. ○ Disk; △ 150° cone; ▽ 120° cone; ■ 90° cone; ▶ 60° cone.

was about 75 °F, and the corresponding Schmidt number for naphthalene is 2·4. From the calculation presented by Sparrow & Gregg, C is given as 0·625. Thus the predicted laminar mass-transfer relation in the present investigation is

$$\overline{Sh} = 0{\cdot}625 Re_0^{0\cdot 5}.$$

The experimental mass-transfer results for the rotating disk (i.e. $\alpha = 90$ deg.) are shown in figure 2 as well as results obtained by Kreith *et al.* for a similar experimental model. The results of these two experiments in the laminar range are in good agreement with each other and also with the theoretical prediction. The onset of flow transition differs slightly in the two investigations, probably because of differences in the condition of the naphthalene surfaces. In the investigation of Kreith *et al.* the molten naphthalene was cast onto the aluminium disk, while in the present work it was applied to the disk with a brush. The difference in procedure gives the surface a slightly different texture.

The average mass-transfer results from the various cones are presented in figure 3. The experimental results in the laminar range from the 180°, 150°, 120°, and 90° cones are in good agreement with the predicted results. The exact point of flow transition differs between models, but it is reasonably consistent in view of the non-uniformity of different cone surfaces. The mass-transfer results from the 60° cone, however, exhibit quite a different characteristic as shown in figure 3. The cause of this different characteristic is believed to be the breakdown of the boundary-layer behaviour in the flow over the cone surface. The theoretical analyses of Wu and Tien on the flow and the heat transfer about rotating cones are limited by the boundary-layer assumption. As the vertex angle of the cone becomes smaller, the large curvature of the surface and the large centrifugal force acting normal to the surface render the boundary-layer assumption invalid. No attempt has been made to determine the smallest vertex angle which will permit a boundary-layer flow, as the breakdown of the boundary layer is physically a gradual process. It should be emphasized here that the present results do not indicate conclusively the breakdown of the boundary-layer behaviour on a heated metal cone with a vertex angle of 60°. But they show only that the boundary layer on the sublimation model breaks down.

As the flow behaviour is different in the case of the 60° cone, a different flow-transition characteristic is expected and is qualitatively confirmed as will be discussed later.

Flow-transition characteristics

The region upstream of flow transition includes two types of flow. The inner core consists of plain laminar flow, while the outer ring is a region of instability bounded by the radii of instability and transition. The work of Gregory, Stuart & Walker (1955) demonstrates theoretically that many different modes of instability are possible in the three-dimensional boundary layer on a rotating disk. Most of the disturbances consist of progressive wave patterns, but at any station in the flow, there is certainly one mode of disturbance which is stationary relative to the surface. When such a disturbance appears, the result is a pattern of fixed disturbance vortices.

In the present investigation, an attempt was made to measure the onset of instability for the various cones. The cones were rotated for a relatively long period of time to permit a vortex pattern to be traced on the cone surface in the region of instability. Unfortunately, the naphthalene surface does not afford as clear an indication of the onset of instability as desired. However, measurements made with the naphthalene surface on the 90°, 120°, 150° cones and the disk indicate that the vortex pattern used on the cone surface starts at a Reynolds number in the vicinity of 180,000 for the above-mentioned cone models. This value of the onset of instability is in fair agreement with those obtained by Gregory & Walker (1960), who found values of about 200,000 for a polished disk, 140,000 for a disk with a slotted surface, and 100,000 for a wire-cloth surface.

The fact that the cone surface was not polished, and in addition, the effect of naphthalene being injected into the boundary layer, would cause instability to occur at a lower Reynolds number than for a polished metal disk.

The characteristic tracings of the vortex system as observed in the region of instability over the other cones were never present on the 60° cone, and attempts to create these tracings by inducing surface disturbances were unsuccessful. The 60° cone consistently developed, except near the vertex, a higher polish on the naphthalene coating than was ever developed on the other cones. These observations suggest qualitatively that the flow and the transition characteristics over the 60° cone are different.

The critical Reynolds number, Re_c, which is based on the radius of transition, is found from the mass-transfer measurements to be in the range of Reynolds number from 200,000 to 250,000 for cones of vertex angles of 180°, 150°, 120° and 90°. In the case of the 60° cone the flow transition is not as clearly indicated in the mass-transfer measurements as that of other cones. The mass-transfer results of the 60° cone, however, do show a change of relationship with the Reynolds number in the vicinity of the Reynolds number 100,000, which is about one-half of the critical Reynolds number for all other cone models.

Mass-transfer results in turbulent range

The turbulent heat transfer from a rotating disk was first studied experimentally by Cobb & Saunders. They also presented some semi-empirical formulas based on the friction analogy. An improved analogy calculated by Kreith *et al.* based on a critical Reynolds number of 200,000 is shown in figure 2 with all the experimental results of Cobb & Saunders. In the limiting case in which all the disk surface is covered by turbulence, Davies obtained an approximate solution as

$$\overline{Nu} = KPrRe_0^{0\cdot8},$$

where K is a constant given numerically by Davies as 0·0195. For air ($Pr = 0{\cdot}72$), KPr is 0·014, which agrees with the value of 0·015 suggested by Cobb & Saunders based on the friction analogy. The turbulent mass-transfer measurements are also presented in figure 2 along with the analogy calculations by Kreith *et al.* and the solution based on the Davies theory for the limiting case of complete turbulent flow.

There appear no theoretical or semi-empirical attempts in the literature at calculating the turbulent heat or mass transfer for rotating cones other than for the special geometry of a disk. The mass-transfer measurements presented in figure 3, however, indicate that the analogy calculation of Kreith *et al.* and the approximate solution of Davies for a rotating disk can be applied as well to cones of large vertex angles. Even in the case of the 60° cone, the calculated results are in good agreement with the measurements at high rotational Reynolds numbers.

Also shown in figure 3, a single functional relationship between the measured Sherwood number and the rotational Reynolds number exists as in the laminar case for cones of vertex angles of 180°, 150°, 120° and 90°. This again suggests that the turbulent flow and the mass-transfer mechanisms are similar in these cases. The larger Sherwood number for the 60° cone in the range of the Reynolds number from 100,000 to 400,000 is probably due to the early flow transition at the Reynolds number 100,000. This early transition gives a larger portion of the surface covered by turbulent flow and consequently a higher mass-transfer coefficient. At high Reynolds numbers the increased portion, due to early transition, as compared to the portion covered by turbulent flow becomes small and the increase of mass-transfer coefficient is negligible as also shown in figure 3.

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